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MATHEMATICAL DESCRIPTION OF THE PROCESS OF MINERAL-FIBER FORMATION BY THE PLASMA METHOD

A. N. Laktyushin and V. L. Sergeev

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The problem of microfiber formation from a melt jet of mineral raw material in exposure to a flow of electric-arc plasma is formulated. Its solutions are given for some simple cases.

One of the likely mechanisms of fiber formation from a melt of mineral raw material with the aid of an air plasma jet is disturbance of the melt surface and pulling of elementary jets from waves and droplets, which are then cooled to solidification.

The goal of the present work is to describe mathematically and calculate the process of fiber formation.

The stage of material melting to obtain fiber using a plasma jet is an independent problem and has been investigated earlier in [1].

Below, consideration is given to an elementary melt jet with initial diameter d_0 in a longitudinal gas flow, which owing to friction accelerates and stretches the jet. Simultaneously, the melt is cooled to solidification and fiber formation.

The active force that causes pulling of an elementary jet is the force of aerodynamic friction of the concurrent gas flow F_{aero}. This force must overcome the viscosity forces F_{vis} occurring in the melt jet, the inertia forces F_{iner}, and the surface-tension forces F_{ten}. In the case of vertical pulling, the gravitational force F_{grav} can be considerable.

Thus, in the general case the following balance of forces must exist:

$$F_{\text{aero}} + F_{\text{vis}} + F_{\text{iner}} + F_{\text{ten}} + F_{\text{grav}} = 0 . \tag{1}$$

In horizontal pulling of thin fibers the gravitational force is insignificant.

The first three terms in Eq. (1), as the basic forces determining fiber pulling, are reported in [2-4]. Therefore, expression (1) is reduced to

$$F_{\text{aero}} + F_{\text{vis}} + F_{\text{iner}} = 0 . \tag{1a}$$

As noted in [2], the surface-tension term "can be neglected in the technology of fiber formation." At low rates of melt efflux from the draw plate, the maximum drop diameter is determined [3] by the surface-tension forces:

$$d_{\text{d}} = (6\sigma/\rho g)^{1/3}\, d_{\text{d.p}}^{1/3} . \tag{2}$$

The time of existence of a liquid thread up to its rupture [3] is

$$\tau = (3\eta/2\sigma)\, d_{\text{th}} . \tag{3}$$

The magnitude of the surface tension can be considerable for glass in the initial stage of the process [3].

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The melt flow in an elementary jet is determined by motion and continuity equations known from hydrodynamics.

Application of Newton's law to a stream in the flow leads to the equation of motion [5]

$$\Sigma F_i = \rho dV\, DW/d\tau\,, \tag{4}$$

where ΣF_i is the sum of the forces acting on the volume element dV; $DW/d\tau$ is the acceleration; ρdV is the mass.

In the general case, the equations of motion for W_z and of continuity are presented in cylindrical coordinates in A. V. Luikov's monograph [6]:

$$\frac{dW_z}{d\tau} + W_r \frac{dW_z}{dr} + W_z \frac{dW_z}{dz} = -g_z - \frac{1}{\rho}\frac{dP}{dz} + \nu \left(\frac{d^2 W_z}{dr^2} + \frac{d^2 W_z}{dz^2} + \frac{1}{r}\frac{dW_z}{dr} \right), \tag{5}$$

$$\frac{dW_r}{dr} + \frac{dW_z}{dr} + \frac{W_r}{r} = 0\,. \tag{6}$$

The case under consideration has its special features: the gravitational force and the pressure drop along z in a free jet can be neglected, but a new force, namely, the aerodynamic friction of the gas flow against the melt jet, appears. Moreover, as mentioned above, the flow considered is steady. For the force of friction of the melt layers over the radius Newton's law $\tau = \mu(dW_z/dr)$ is adopted [5], and therefore the term d^2W_z/dz^2 in Eq. (5) can be neglected [7].

We assune that the melt density ρ_f is constant and the viscosity depends on the temperature and, consequently, on the longitudinal coordinate z.

The aerodynamic-friction force F_{aero} is equal to [2, 4]

$$F_{aero} = c_f \frac{\rho_g W_{rel}^2}{2} S\,, \tag{7}$$

where $W_{rel} = W_g - W_f$ is the relative velocity of the fiber in the gas flow; c_f is the coefficient of resistance, $c_f = 0.65R^{-0.7}$ for the turbulent boundary layer on a cylinder surface [2].

With account for (1a) and the above explanations the equation of motion in the melt jet for the case of steady-state flow will have the form

$$W_z \frac{dW_z}{dz} = \frac{\nu(z)}{r}\frac{d}{dr}\left(r\frac{dW_z}{dr} \right) + c_f \frac{\rho_g}{\rho_f}\frac{W_{rel}^2}{2}\frac{2}{R(z)}, \tag{8}$$

where the first term corresponds to F_{iner} in (1a), the second term, to F_{vis}, and the third, to F_{aero}.

For a one-dimensional steady-state flow in a channel of variable cross section (and in a jet) at ρ = const the continuity equation [6] can be written as [5]

$$W_z \frac{dS_z}{dz} + S_z \frac{dW_z}{dz} = 0\,, \tag{9}$$

whence [2]

$$S_z W_z = \text{const}\,,\ \ R^2(z)\, W_z = \text{const}\,, \tag{10}$$

or

$$G = \rho S_z W_z = \text{const}\,, \tag{10a}$$

where $S_z = \pi R_z^2$ is the cross-sectional area of the melt jet.

Sometimes the volume V is assumed to be constant (the law of mass conservation at ρ = const) in fiber extension: $V_0 = V = \pi R_z^2 z$ [2, 8].

The energy equation in cylindrical coordinates is [5]

$$\frac{dt}{d\tau} + W_r \frac{dt}{dr} + W_\varphi \frac{dt}{d\varphi} + W_z \frac{dt}{dz} = a \left(\frac{d^2 t}{dr^2} + \frac{1}{r} \frac{dt}{dr} + \frac{1}{r} \frac{d^2 t}{d\varphi^2} + \frac{d^2 t}{dz^2} \right), \tag{11}$$

where a is the thermal diffusivity.

In accordance with [8] the difference in the temperatures on the surface and along the axis of typical glass fiber samples in their cooling does not exceed several percent. Therefore it can be assumed that the fiber temperature during cooling is constant over its cross section. Obviously, this condition is valid only for sufficiently thin fibers at moderate cooling rates. With account for the above assumption of one-dimensional steady-state flow Eq. (11) is reduced to

$$W_z \frac{dt}{dz} = a \frac{d^2 t}{dz^2}. \tag{12}$$

With an insignificant temperature difference over the radius, for a jet element dz the balance equation of heat losses in cooling in a concurrent gas flow can be written as

$$Q d\tau = G c_p dt = \alpha (t_g - t) S_{\text{side}} d\tau , \tag{13}$$

where $G = V\rho = (\pi d^2/4)\rho dz$; $S_{\text{side}} = \pi d dz$; α is the heat-transfer coefficient, determined, for instance, by the criterial expression [9]

$$\mathrm{Nu} = 0.325 \mathrm{Re}^{0.3} . \tag{14}$$

Solving (13), we arrive at

$$\vartheta/\vartheta_0 = \exp(-4\alpha\tau/dc\rho) , \tag{15}$$

where $\vartheta = t_0 - t_g$; t_0 is the initial temperature.

Combining energy equation (12) written for the unsteady-state case

$$\frac{dt}{d\tau} + W_z \frac{dt}{dz} = a \frac{d^2 t}{dz^2}, \tag{16}$$

with (13) in the form

$$\frac{dt}{d\tau} = -\frac{4\alpha\vartheta}{dc\rho}, \tag{17}$$

we obtain the energy equation for the melt jet

$$-\frac{4\alpha\vartheta}{dc\rho} + W_z \frac{d\vartheta}{dz} = a \frac{d^2\vartheta}{dz^2}. \tag{18}$$

Determination of an exact quantitative relation between the main parameters in interaction of an energy carrier with an elementary melt jet "seems to be impossible" [4] because of numerous factors that change over the length and radius of a fiber. Therefore it is reasonable to have approximate solutions of the problem that can be used to describe the process of fiber formation.

The problem of fiber formation is reduced to simultaneous solution of the equations of motion (5), continuity (10), and energy (12) or (18) written as applied to an elementary melt jet for corresponding initial, boundary, and final conditions. In particular, the main final condition for the process is the solidification temperature of the melt jet, when fiber pulling and reduction of its diameter cease.

Let us consider solutions of the system of equations (5), (10), (17), (18) for some simple cases.

It is known that in melt efflux from a draw plate the velocity distribution in the melt jet with respect to the radius is of parabolic form until it flattens out over the radius [2]. In this case, $(dW_z/dr) = 0$ and the equation of motion is simplified to

$$W_z \frac{dW_z}{dz} = c_f \frac{\rho_g}{\rho_f} \frac{W_{rel}^2}{R(z)}, \tag{19}$$

where $W_{rel} = W_g - W_z$.

In cases where it can be assumed that $dW_z/dz = 0$, from Eq. (19) it follows that

$$W_g = W_z. \tag{20}$$

From continuity equation (10) in the form $\pi R_z^2 z = V_0 = \text{const}$

$$R_z = \sqrt{\left(\frac{V_0}{\pi z}\right)}. \tag{21}$$

Substituting (20) into energy equation (12) and solving the latter, we obtain

$$t = t_0 \exp\left(-\frac{W_g}{a} z\right). \tag{22}$$

Formulas (21) and (22) are rather approximate since they are based on rough assumptions, but nevertheless they show the character of the dependence of the fiber dimensions on the gas flow velocity, the volume of the initial fiber formed, and the initial temperature and thermal diffusivity of the melt.

Now we consider the case where it can be assumed that $dW_z/dr = 0$, $dW_z/dz \neq 0$. Replacing R_z in Eq. (19) by its value from (21), we obtain an equation from which the distribution of W_z with respect to z can be found.

In particular, if $W_z \ll W_g$, then

$$W_z \frac{dW_z}{dz} = A\sqrt{z}, \tag{23}$$

where $A = c_f(\rho_g/\rho_f)(W_g^2/\sqrt{V_0/\pi})$, whence

$$W_z = \sqrt{\left(\frac{4}{3} A z^{3/2} + W_0^2\right)} \tag{24}$$

(W_0 is the jet velocity at $z = 0$).

Substituting (24) (let $W_0 = 0$) into energy equation (12), we obtain

$$A_1 W_g z^{3/4} \frac{dt}{dz} = a \frac{d^2 t}{dz^2}, \tag{25}$$

where $A_1 = 2\sqrt{(\sqrt{\pi}/(3\sqrt{V_0}))c_f(\rho_g/\rho_f)}$, whence

$$t = \left(\frac{dt}{dz}\right)_0 \left(\int \exp\left(A_2 W_g z^{7/4}\right) dz\right) + t_0, \tag{26}$$

($A_2 = 4A_1/7a$, t_0 and $(dt/dz)_0$ are the temperature and its derivative at $z = 0$).

One more simple solution of the problem can be obtained if the energy equation in the form (18) is used in combination with the solution of equation of motion (20).

Assuming the following relation for the temperature distribution with respect to the length:

$$\vartheta = \vartheta_0 \exp(-A_3 z) \tag{27}$$

and substituting the value of ϑ of (27) into (18), we arrive at an equation for determination of the coefficient A_3:

$$aA_3^2 + W_g A_3 + \frac{A\alpha}{c\rho d} = 0 , \tag{28}$$

whence

$$A_3 = \left(-W_g + \sqrt{\left(W_g^2 - 4a\frac{4\alpha}{c\rho d}\right)}\right) / 2a . \tag{29}$$

Thus, having determined the fiber length z_f from (27) by the temperature of pulling cessation, we can find the final radius of fiber from formula (21) (or (10a)).

From (17) we have for t

$$t = t_0 - \frac{4\alpha (t_0 - t_g) \exp(-A_3 z)}{c\rho d} \tau . \tag{30}$$

One more expression for the temperature field in a fiber can be obtained by substituting the value of the fiber radius (21) into (13):

$$t = t_g + (t_0 - t_g) \exp\left(-\frac{2\alpha}{c\rho}\sqrt{\left(\frac{\pi z}{V_0}\tau\right)}\right) . \tag{31}$$

Expressions (30) and (31) give the temperature distribution with respect to the fiber length as a function of the initial melt temperature, the heat-transfer conditions (α, t_g, W_g), the thermophysical properties of the fiber ($c\rho$ and a), and the time.

As a whole, the formulas obtained allow evaluation of the fiber dimensions and the temperature field in it as a function of the main parameters of the process.

NOTATION

η, dynamic viscosity, $N \cdot sec/m^2$; t, temperature, °C; ϑ, excess temperature, K; τ, time, sec; ν, kinematic viscosity, m^2/sec; ρ, density, kg/m^3; σ, surface tension, N/m; d, diameter of the fiber or the melt jet, m; g, gravitational acceleration; Nu, Nusselt number; Re, Reynolds number; A, constant; r, radius of the jet in the flow, m; z, dimensionless coordinate; c, heat capacity, J/(kg·K); W, dimensionless velocity; R, radius of the region of plasma- jet action on the fiber surface, m; V, volume, m^3; G, mass flow rate, kg/sec; S, lateral surface of the fiber, m^2; Q, amount of heat, kJ/sec. Subscripts: g, gas; 0, initial quantity; f, fiber.

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